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On large angle multiple gluon radiation

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Abstract: Jet shape observables which involve measurements restricted to a *part* of phase space are sensitive to multiplication of soft gluon with large relative angles and give rise to specific single logarithmically enhanced (SL) terms (non-global logs).
We consider *associated* distributions in two variables which combine measurement of a jet shape V in the whole phase space (global) and that of the transverse energy flow away from the jet direction, $E_{\rm out}$ (non-global).
We show that associated distributions factorize into the global distribution in V and a factor that takes into account SL contributions from multi-gluon "hedgehog" configurations in all orders. The latter is the same that describes the single-variable $E_{\rm out}$ distribution, but evaluated at a rescaled energy VQ.

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1. Introduction

In a theory with dimensionless coupling one would naively expect cross sections to scale in a simple manner at very large Q^2, $Q^2 \gg m^2$, with m a generic mass scale. However, in QCD this is not true for two reasons. Firstly, due to UV divergences, the effective interaction strength does vary with scale (as in any quantum field theory with dimensionless coupling). As a result perturbative (PT) corrections to cross sections slowly decrease with Q^2 as powers of $\alpha_{\rm s} \propto 1/\ln(Q^2/\Lambda^2_{\rm QCD})$. Secondly, there are, generally speaking, collinear and infrared divergences (collinear 2-parton splittings, soft gluon radiation). As well known (Bloch-Nordsieck [1]), in "good" inclusive observables which do not include observation (fixing the momentum) of a single hadron, either in the initial or final state, the logarithmic collinear and infra-red divergences cancel.

The classical examples are the total cross section of e^+e^- annihilation into hadrons, $\tau \to$ hadrons decay width. Here the collinear and infra-red divergences, present in both real and virtual corrections, cancel completely in the (unrestricted) sum. Since here we don't have dimensional parameters other than Q^2 (in the $m^2/Q^2 \to 0$ limit), the total cross section is given by the simple Born expression modulo calculable $\alpha_{\rm s}^n(Q^2)$ corrections.

$$\sigma(Q) = \sigma^{\rm Born}(Q) \cdot g\left(\alpha_{\rm s}(Q^2)\right), \qquad g(0) = 1 \qquad (Q^2 \gg m^2)\,.$$

Moving to *less inclusive* measurements one faces different situations. The first case involves fixing (measuring) momentum of a hadron, e.g. that of the initial proton in DIS, DY, ... (structure functions, SF) or of a final hadron (fragmentation function). Then soft divergences still cancel but collinear ones do not, making such observables not calculable at the parton level. These effects, however, turn out to be universal and, given a proper technical treatment, can be *factored out* as non-perturbative (NP) inputs. What remains under control then is only the Q^2-dependence (scaling violation pattern).

Collinear divergences in the *final* state may be avoided altogether if one looks at *energy flows* rather than individual hadrons. More generally, one can study a family of the so-called collinear and infra-red safe (CIS) jet shapes V,

$$V = \sum_i v(k_i)\,, \tag{1.1}$$

where the sum runs over all particles (hadrons) in the final state, $v(k)$ being a contribution of a single particle with 4-momentum k. Being (by construction) *linear* in particle momenta, such observables are also free from collinear (and soft) *divergences*.

However, here the cancellation of real and virtual effects is not complete and leaves trace in the PT-calculable distributions over V. Indeed, taking the value of a generic jet shape observable $V \ll 1$ we squeeze the phase space thus inhibiting *real* parton production and multiplication. Since the *virtual* PT radiative contributions remain unrestricted, the *divergences* do cancel but produce finite but logarithmically enhanced leftovers:

$$\Sigma(Q,V) \equiv \int_0^V dV \frac{d\sigma(V)}{\sigma_{\rm tot}\, dV} = f\left(\alpha_{\rm s}(Q^2), \ln V\right). \tag{1.2}$$

Each gluon emission brings in at most two logarithms (one of collinear, another of infra-red origin). These leading contributions are due to multiple soft gluon bremsstrahlung off the primary hard partons that form the underlying event, which can be looked upon as independent gluon radiation and can be easily treated. A clever reshuffling of PT series, based on universal nature of soft and collinear radiation (factorization) results in the *exponentiated* answer in the form

$$\ln \Sigma(Q,V) = \sum_{n=1}^{\infty} \alpha_{\rm s}^n(Q^2) \left(A_n \ln^{n+1} V + B_n \ln^n V + \cdots\right). \tag{1.3}$$

In what follows we will discuss only these two series of term and neglect subleading small corrections of the order of $\alpha_{\rm s}$, $\alpha_{\rm s}^2 \ln V, \ldots$. The A_n series is referred to as double logarithmic (DL) and B_n as single logarithmic (SL).

The whole leading series A_n actually originates from the effect of running of the coupling in the basic one gluon radiation term, $\alpha_{\rm s} \ln^2 V$, $n = 1$. Indeed, the expansion in $\alpha_{\rm s}(Q^2)$ is pretty artificial since in reality it is a wide range of scales, $(VQ)^2 \ll k^2 \ll Q^2$, at which the coupling $\alpha_{\rm s}(k^2)$ actually enters:

$$A_1 \alpha_{\rm s}(Q^2) \ln^2 V \implies A_1 \int_{(VQ)^2}^{Q^2} \frac{dk_t^2}{k_t^2}\, \alpha_{\rm s}(k_t^2) \ln \frac{Q}{k_t} = \sum_{n=1}^{\infty} A_n \cdot \alpha_{\rm s}^n(Q^2) \ln^{n+1} V\,, \tag{1.4}$$

where k_t is the gluon transverse momentum and the logarithmic factor is due to the soft enhancement. As a result, the coefficients A_n are straightforward to obtain. It is important to realise that what makes the coupling run in minkowskian observables is collinear gluon splittings (into gluons with energies of the same order, "hard splitting", and $q\bar{q}$ pairs) and the inclusive CIS nature of the observable (see, e.g. [2]).

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Let us remark in passing that the scales *below* $(VQ)^2$ with k_t^2 running down to zero are actually present as well. They, however, do not contribute at the PT level but give rise to the NP power suppressed corrections (see [2, 3]).

The subleading series B_n start from the first SL correction $\alpha_s \ln V$ $(n = 1)$ (of either collinear or soft origin). In higher orders, $n \geq 2$, taking care of SL terms involves a careful treatment of α_s (physical scheme), of its running argument, as well as precise fixing of the scales of the leading logarithmic terms in (1.3).

In the case of the so-called "global" observables, that is measurements in which the observable (1.1) accumulates contributions from final state particles in the whole phase space, all SL contributions are generated by a careful treatment of gluon bremsstrahlung off the primary partons (Sudakov exponentiation). Obviously, there exists also gluon multiplication and, in particular, gluon bremsstrahlung off *secondary* partons. As already stated, *collinear* "hard" gluon decays make the coupling run. As for *soft* gluon bremsstrahlung off secondary gluons, it does not affect global observables at the DL+SL accuracy (1.3) (the first correction being $\mathcal{O}\left(\alpha_s^2 \ln V\right)$). Global observables have been intensively studied in the literature both perturbatively [4, 5] (DL+SL as in (1.3)) and with account of the first leading NP power correction $\mathcal{O}\left(1/Q\right)$ [3].

However, as was recently noticed by Dasgupta and Salam, certain jet shape measurements turn out to be sensitive to soft gluon multiplication effects at the SL level starting from $n = 2$. These observables involve measurements restricted to a *part* of phase space and were correspondingly dubbed *non-global* [6]. The simplest example is given by particle transverse energy flow $E_{\rm out}$ in e^+e^- annihilation, measured in an angular (pseudorapidity) region $C_{\rm out}$ which is away from the jet direction (thrust axis) by a finite angle θ_0:

$$\Sigma_{\rm out}(Q, E_{\rm out})\,, \qquad E_{\rm out} = \sum_{i \in C_{\rm out}} k_{ti}\,. \tag{1.5}$$

It is a CIS observable. Moreover, there is obviously no collinear enhancement $(A_n \equiv 0)$. The leading effect is SL — exponentiation of independent large angle soft gluon emission off the primary jets. Apart from it, however, additional contributions of the same order emerge here due to soft gluon-gluon correlations, which were absent in global observables.

Imagine a system of gluons whose energies are strongly ordered:

$$k_\ell \ll k_{\ell-1} \ll \cdots \ll k_1 \ll p \simeq \frac{Q}{2}\,.$$

Then only the *hardest* of the gluons belonging to $C_{\rm out}$ would contribute to the observable while the contributions of all other (much softer) gluons would cancel against corresponding virtual corrections. This means that it suffices to consider multi-gluon systems with the *softest* offspring $k_n \in C_{\rm out}$ being the only one to be measured, while all harder companions do not contribute to the observable, $k_i \in C_{\rm in}$, $i < n$, where $C_{\rm in}$ is the complementary angular region, close to the jets.

Such configurations of $n \geq 2$ gluons contribute to the integrated distribution (1.5) at the SL level as $(\alpha_s \ln(Q/E_{\rm out}))^n$. This is the origin of the Dasgupta-Salam discovery.

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These specific subleading contributions are not easy to analyse analytically order by order. In spite of the fact that in the strong energy ordered kinematics all $(n+2)!$ amplitudes are known and given by soft insertion rules, to give a compact answer to the *square* of the n-gluon matrix element is possible only in the large-N_c limit (see [7]). Moreover, angles between gluons are of the order one (hedgehog configurations), so that the phase space integrations can be handled only numerically. As we shall show below, the all-order resummation of these contributions can be carried out in the large-N_c limit. This leads to an evolution equation [8] which has a highly non-linear structure. Therefore, its solution can be given in an analytic form only in an academic high-Q limit ($\alpha_s \ln(Q/E_{\rm out}) \gg 1$).

Recall that non-linear evolution equations for generating functionals describing multi-parton ensembles have a long history. In the leading *collinear* approximation they, in particular, form the basis for Monte Carlo generators and for the standard theoretical jet studies (jet rates, in-jet particle multiplicities and spectra, etc.) [9, 7]. When large angle soft gluon radiation becomes important (as, e.g., in inter-jet particle flows — the bread for the so-called string effect), in the *collinear* approximation (that is when other secondary gluons stay quasi-parallel to the primary hard parton) the QCD coherence is at work. As a result, a single soft gluon emission at large angles is very simple and is determined by the total colour charge of the jet.

However, when the large-angle hedgehogs are considered, the new evolution equation for such systems has an essentially different (and more complicated) structure which, as mentioned above, can be practically handled only in the large-N_c approximation.

It is important to stress that most of the actual experimental measurements are subject to non-global effects. Experiments often (if not always) involve phase space restriction. For example,

- the very first CIS jet cross section invented by Sterman and Weinberg back in 1979 [10] provides a perfect example of a non-global observable;
- in hadron-hadron interactions accompanying hadron production is studied in a limited rapidity range (in particular, the famous *pedestal* distributions in hard hadron-hadron collisions);
- direct photon studies necessarily involve photon isolation criteria;
- a family of QCD string/drag effects deals with particle flows in restricted inter-jet angular regions;
- profiles of a separate jet (rather than studying shape variables of the whole event). For example, characteristics of the current fragmentation jet in DIS which is based in a one-hemisphere particle selection.

All these and many other similar measurements contain non-global PT corrections. With exception of the current jet in DIS (see [11]) and the case study of the $E_{\rm out}$ distribution in e^+e^- [12, 8, 13], the non-global effects have not been studied theoretically even at level of the first $(\alpha_s \log)^2$ correction.

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In spite of being formally subleading, these effects may significantly modify the PT predictions for the non-global observables, as was shown by the case study of the $E_{\rm out}$ distribution in e^+e^- by Dasgupta and Salam [12].

Berger, Kucs and Sterman [14] have recently formulated a programme of how to avoid non-global logarithms in a measurement of transverse energy flow away from jets ($E_{\rm out}$). They suggested to squeeze the jets and thus suppress multi-gluon hedgehogs in the $C_{\rm in}$ region. In e^+e^- this amounts to introducing the *associated* distribution in two variables, $E_{\rm out}$ and V (for example, $V = 1-T$ with T the thrust), and considering the region $V \ll 1$ which selects 2-jet-like configurations,

$$\Sigma_{\rm 2ng}(Q, V, E_{\rm out})\,.$$

They treated the region in which the two characteristic scales of the problem are comparable, $VQ/E_{\rm out} = \mathcal{O}(1)$, so that

$$\alpha_{\rm s} \ln \frac{VQ}{E_{\rm out}} \tag{1.6}$$

amounts to a negligible correction $\mathcal{O}(\alpha_{\rm s})$. The authors stated in the Conclusions to [14] that in this approximation their "formalism is sensitive mainly to radiation stemming directly from primary hard scattering".

In this paper we consider a general case[1] of $E_{\rm out}$ being potentially much smaller than VQ. We show that, having extracted the global DL and SL enhanced term in $\ln V$, the remaining SL corrections $(\alpha_{\rm s}\ln V)^n$ and $(\alpha_{\rm s}\ln(Q/E_{\rm out}))^n$ conspire to produce the powers in (1.6).

We analyse these non-global logarithms in all orders and demonstrate that the associated distributions factorize as

$$\Sigma_{\rm 2ng}(Q, V, E_{\rm out}) = \Sigma(Q, V)\cdot \Sigma_{\rm out}(VQ, E_{\rm out})\,. \tag{1.7}$$

Here $\Sigma(Q, V)$ is the standard global distribution (1.2) and $\Sigma_{\rm out}(Q, E_{\rm out})$ is the SL distribution in $E_{\rm out}$ (at the same total energy Q) which takes into account contributions from multi-gluon hedgehog configurations in all orders and contains the full dependence on the geometry of the measurement (θ_0). This is the main result of the paper.

We will also discuss the asymptotic behaviour which according to (1.7) reduces to that of the simplest non-global distribution $\Sigma_{\rm out}$ which had been studied in [8].

Let us stress that one needs to keep under the best possible control effects induced by radiation of relatively soft gluons not merely for the sake of improving PT predictions. What makes such studies even more important and interesting is the fact that the physics of small transverse momentum gluons is that of confinement.[2]

[1] We take V to be a global observable, though one can equally well restrict V to $C_{\rm in}$ as was done in [14].
[2] In market terms, a \$$10^6$ problem [15]

2. Observables

In this paper we will consider e^+e^- annihilation into hadrons and define a cone around the thrust axis and denote by $C_{\rm out}$ and $C_{\rm in}$ the regions outside and inside the solid cone (inter and intra jet regions).

We start by considering the first factor in (1.7) that is the (integrated) *global* distribution $\Sigma(Q,V)$ defined as

$$\Sigma(Q,V) = \sum_n \int \frac{d\sigma_n}{\sigma_{\rm tot}} \Theta\left(V - \sum_{i\in C_{\rm tot}} v(k_i)\right), \tag{2.1}$$

with the sum extended to the full phase space $C_{\rm tot} = C_{\rm in} + C_{\rm out}$. Here $d\sigma_n$ and $\sigma_{\rm tot}$ are the production and total cross sections. The probing function $v(k)$, linear in the particle momentum k, can be represented as

$$v(k) = \frac{k_t}{Q} \cdot h(\eta)\,, \tag{2.2}$$

with k_t the transverse momentum with respect to the thrust axis and η the pseudorapidity. To get the distribution in thrust, T, we have to take $h(\eta) = e^{-|\eta|}$ and the set $V = 1-T$. For small $1-T$, summing over final particles we may neglect contributions from the primary hard partons (quarks) since $h(\eta)$ vanishes at large $|\eta|$, and restrict ourselves to to considering only secondary soft gluons.

The distribution in broadening B, for example, corresponds to $h(\eta) = 1$, $V = 2B$. In this case, even for $B \ll 1$, one needs to include the contributions from the recoiling primary quarks.

Now we define analogously the *non-global* distribution (the second factor on the r.h.s. of (1.7)):

$$\Sigma_{\rm out}(Q,E_{\rm out}) = \sum_n \int \frac{d\sigma_n}{\sigma_{\rm tot}} \Theta\left(E_{\rm out} - \sum_{i\in C_{\rm out}} k_{ti}\right), \tag{2.3a}$$

where we have chosen to measure the transverse energy flow accumulated in the angular region $C_{\rm out}$ away from the jets.

Finally, as was suggested by Berger, Kucs and Sterman [14], we introduce the shape observable distribution in two variables (the *associated* distribution)

$$\Sigma_{\rm 2ng}(Q,V,E_{\rm out}) = \sum_n \int \frac{d\sigma_n}{\sigma_{\rm tot}} \Theta\left(V - \sum_{i\in C_{\rm tot}} v(k_i)\right) \cdot \Theta\left(E_{\rm out} - \sum_{i\in C_{\rm out}} k_{ti}\right). \tag{2.3b}$$

3. Analysis and resummation

We are interested in the soft region

$$V, E_{\rm out} \ll Q\,. \tag{3.1}$$

3.1 Mellin factorization

First we recall how the resummation with the SL accuracy is done for global observables. To this end one has to factorize the theta-function in (2.1) by the Mellin transform:

$$\begin{aligned}\Sigma(Q,V) &= \int \frac{d\nu}{2\pi i\nu}\, e^{\nu V}\, \tilde{\Sigma}(Q,\nu)\,,\\ \tilde{\Sigma}(Q,\nu) &= \sum_n \int \frac{d\sigma_n}{\sigma_{\rm tot}} \prod_{i\in C_{\rm tot}} u(k_i)\,, \qquad u(k)=e^{-\nu\, v(k)}\,. \end{aligned}\tag{3.2}$$

As has been mentioned above, in the thrust case the contributions of the primary partons can be dropped (since $v(k)$ vanishes at large $|\eta|$).

In the case of broadening the recoiling primary partons do contribute. This causes certain complication. This recoil effects can be taken care of by expressing the momenta of recoiling quarks in terms of those of the secondary partons via 3-momentum conservation. Thus, also in the case of broadening one can restrict the product in (3.2) to the secondary soft partons, provided one factorizes also the momentum conservation constraints by the Fourier transform. This amounts to incorporating into $\tilde{\Sigma}$ an additional Fourier variable associated with transverse momentum conservation. Peculiarities of B as a variable employed to squeeze the jets, can be analyzed along the lines of [5]. This leads to the known modification of the *global* distribution but will not affect the dependence on $E_{\rm out}$ in (1.7).

In what follows to simplify the discussion we neglect this complication and concentrate on the thrust case.

For the out-distribution we have

$$\begin{aligned}\Sigma_{\rm out}(Q,E_{\rm out}) &= \int \frac{d\nu_{\rm out}}{2\pi i\nu_{\rm out}}\, e^{\nu_{\rm out}\,E_{\rm out}/Q}\, \tilde{\Sigma}_{\rm out}(Q,\nu_{\rm out})\,,\\ \tilde{\Sigma}_{\rm out}(Q,\nu_{\rm out}) &= \sum_n \int \frac{d\sigma_n}{\sigma_{\rm tot}} \prod_{i\in C_{\rm tot}} u_{\rm out}(k_i)\,,\\ u_{\rm out}(k) &= \theta_{\rm in} + \theta_{\rm out} e^{-\nu_{\rm out}\,k_t/Q} = 1-\theta_{\rm out}\left(1-e^{-\nu_{\rm out}\,k_t/Q}\right). \end{aligned}\tag{3.3}$$

Here $\theta_{\rm in}$, $\theta_{\rm out}$ are the support functions for k in the regions $C_{\rm in}$ and $C_{\rm out}$, respectively:

$$\theta_{\rm in} = \vartheta(|\eta|-\eta_0)\,, \qquad \theta_{\rm out} = \vartheta(\eta_0-|\eta|)\,, \tag{3.4}$$

with η_0 the pseudorapidity corresponding to the opening (half-)angle of the cone,

$$\eta_0 = -\ln\tan\frac{\theta_0}{2}\,.$$

Note that the fact the probing function $u_{\rm out}(k)=1$ for $k\in C_{\rm in}$ means that this parton does not contribute to the measurement.

Finally, for the associated distribution we introduce a double Mellin transform:

$$\begin{aligned}\Sigma_{\rm 2ng}(Q,V,E_{\rm out}) &= \int \frac{d\nu}{2\pi i\,\nu}\frac{d\nu_{\rm out}}{2\pi i\,\nu_{\rm out}}\, e^{\nu V+\nu_{\rm out}E_{\rm out}/Q}\, \tilde{\Sigma}_{\rm 2ng}(Q,\nu,\nu_{\rm out})\,,\\ \tilde{\Sigma}_{\rm 2ng}(Q,\nu,\nu_{\rm out}) &= \sum_n \int \frac{d\sigma_n}{\sigma_{\rm tot}} \prod_{i\in C_{\rm tot}} u_{\rm 2ng}(k_i)\,. \end{aligned}\tag{3.5}$$

The new source function is given by the expression

$$u_{2\mathrm{ng}}(k) = e^{-\nu\, v(k)} \left(\theta_{\mathrm{in}} + \theta_{\mathrm{out}}\, e^{-\nu_{\mathrm{out}}\, k_t/Q}\right) = e^{-\nu\, v(k)} \;-\; \theta_{\mathrm{out}} e^{-\nu\, v(k)} \left(1 - e^{-\nu_{\mathrm{out}}\, k_t/Q}\right). \quad (3.6)$$

It corresponds to measuring V everywhere and E_{out} only in C_{out}.

In the Mellin space all the distributions become exponents of the so-called *radiators*:

$$\tilde{\Sigma}(Q,\nu) \;=\; e^{-R(Q,\nu)}\,, \quad (3.7)$$

$$\tilde{\Sigma}_{\mathrm{out}}(Q,\nu_{\mathrm{out}}) \;=\; e^{-R_{\mathrm{out}}(Q,\nu_{\mathrm{out}})}\,, \quad (3.8)$$

$$\tilde{\Sigma}_{2\mathrm{ng}}(Q,\nu,\nu_{\mathrm{out}}) \;=\; e^{-R_{2\mathrm{ng}}(Q,\nu,\nu_{\mathrm{out}})}\,. \quad (3.9)$$

3.2 Global radiator

For the global distribution the answer is simple and the radiator is known to be given by the SL-improved Sudakov expression

$$R(Q,\nu) = 2C_F \int_0^Q \frac{dk_t^2}{k_t^2}\frac{\alpha_{\mathrm{s}}(k_t^2)}{\pi} \int_0^{\eta_{\max}} d\eta \left(1 - e^{-\nu\, v(k)}\right), \qquad \eta_{\max} = \ln\frac{Q\, e^{-3/4}}{k_t}. \quad (3.10)$$

In the definition of $\eta_{\max}$ we have incorporated a SL correction to the "hard" piece of the quark splitting function

$$\alpha P(\alpha) = \frac{1+(1-\alpha)^2}{2} = 1 - \alpha\left(1 - \frac{\alpha}{2}\right), \qquad \alpha = \frac{k_t}{Q} e^{\eta}.$$

It is straightforward to verify then that the virtual SL contribution due to large rapidities, $\alpha \sim 1$, can be embodied into rescaling of the upper limit of the η integration as follows:

$$\int_{k_t/Q}^{1} d\alpha P(\alpha) \Longrightarrow \int_0^{\eta_{\max}} d\eta\,.$$

The two-loop radiator (3.10) (with the coupling α_{s} in the physical "bremsstrahlung" scheme [16]) has single-logarithmic accuracy. This means that the neglected correction is $\mathcal{O}\left(\alpha_{\mathrm{s}}^2 \ln\nu\right)$ which translates into $\alpha_{\mathrm{s}}^2 \log(Q/V)$ (see, e.g. [5]).

3.3 Non-global radiator

The radiators for both *out* and *associated* non-global distributions, (3.8) and (3.9), have the structure

$$R_A = R_A^{(1)} + R_A^{(c)}\,, \qquad A = \mathrm{out},\, 2\mathrm{ng}\,. \quad (3.11)$$

Here $R_A^{(1)}$ is the standard Sudakov one-gluon emission contribution and $R_A^{(c)}$ is the term due to correlated multi-gluon radiation. (The latter is absent (negligible within the SL accuracy) in the case of a global observable.)

Sudakov piece. The Sudakov piece $R_A^{(1)}$,

$$R_A^{(1)} = 2C_F \int \frac{d^2k_t}{\pi k_t^2}\frac{\alpha_s(k_t^2)}{\pi}\int_0^{\eta_{\max}} d\eta\,[\,1-u_A(k)\,]\,, \tag{3.12}$$

is given by the expression (3.10) with proper probing functions:

$$[\,1-u_{\rm out}(k)\,] = \theta_{\rm out}\left(1-e^{-\nu_{\rm out}k_t/Q}\right), \tag{3.13a}$$

$$\begin{aligned}[\,1-u_{\rm 2ng}(k)\,] &= [\,1-u(k)\,] \,+\, \theta_{\rm out}e^{-\nu v(k)}\left(1-e^{-\nu_{\rm out}k_t/Q}\right)\\ &= [\,1-u(k)\,] \,+\, e^{-\nu v(k)}\,[\,1-u_{\rm out}(k)\,]\,.\end{aligned} \tag{3.13b}$$

We compute first $R_{\rm out}^{(1)}$. Substituting (3.13a) into (3.12) we obtain

$$R_{\rm out}^{(1)}(Q,\nu_{\rm out}) = 2C_F\int_0^Q \frac{dk_t^2}{k_t^2}\frac{\alpha_s(k_t^2)}{\pi}\int_0^{\eta_0} d\eta\left[1-e^{-\nu_{\rm out}\,k_t/Q}\right], \tag{3.14}$$

with the rapidity integral restricted to the $C_{\rm out}$ region. Here we have used that $\eta_0 \sim 1 < \eta_{\max}$. This expression does not have a collinear singularity and therefore is a SL function. Therefore we can approximate $[1-u_{\rm out}]$ in (3.14) by a theta-function $\vartheta(k_t - Q/\nu_{\rm out})$ and evaluate the inverse Mellin integral by simply substituting $Q/E_{\rm out}$ for $\nu_{\rm out}$. We arrive at

$$R_{\rm out}^{(1)}(Q,\nu_{\rm out}) \implies \frac{4C_F}{C_A}\eta_0\cdot\Delta(Q,E_{\rm out}) \,\equiv\, \mathcal{R}_{\rm out}^{(1)}(Q,E_{\rm out})\,, \tag{3.15}$$

where we have introduced a convenient SL function

$$\Delta(Q,E) = C_A\int_E^Q \frac{dk_t}{k_t}\frac{\alpha_s(k_t^2)}{\pi}\,. \tag{3.16}$$

Similarly, using (3.13b) in (3.12) gives the expression for the Sudakov piece of the associated distribution radiator:

$$R_{\rm 2ng}^{(1)}(Q,\nu,\nu_{\rm out}) \;=\; R^{(1)}(Q,\nu) \,+\, \delta R^{(1)}(Q,\nu,\nu_{\rm out})\,. \tag{3.17}$$

Here the first term reconstructs the global distribution $\Sigma(Q,V)$ in (1.7) and the addition contribution $\delta R^{(1)}$ to the Sudakov radiator reads

$$\delta R_{\rm 2ng}^{(1)}(Q,\nu,\nu_{\rm out}) = 2C_F\int^Q \frac{dk_t^2}{k_t^2}\frac{\alpha_s(k_t^2)}{\pi}\int_0^{\eta_0} d\eta\, e^{-\nu\, v(k)}\left(1-e^{-\nu_{\rm out}\,k_t/Q}\right). \tag{3.18}$$

This expression is identical to that for the out-case, (3.14), apart from the additional factor $\exp(-\nu\, v(k))$. Since (3.18) is a subleading SL correction, we can treat this factor, within our accuracy, as simply proving an additional restriction upon the transverse momentum. As long as $\eta = \mathcal{O}(1)$, we have $v(k)\sim k_t$ and thus

$$e^{-v(k)\nu} \quad\Longrightarrow\quad \vartheta(Q-\nu k_t)\,.$$

In V-space this translates into the condition

$$k_t \lesssim VQ \ll Q\,.$$

Then the integral for $\delta R^{(1)}$ becomes the same as that for $R^{(1)}_{\rm out}$ with Q replaced by VQ:

$$\delta R^{(1)}_{\rm 2ng}(Q,\nu,\nu_{\rm out}) \;\Longrightarrow\; \frac{4C_F}{C_A}\,\eta_0\cdot\Delta(VQ,E_{\rm out}) \;=\; \mathcal{R}^{(1)}_{\rm out}(VQ,E_{\rm out})\,. \qquad (3.19)$$

We conclude at the level of the Sudakov contributions to the radiators the factorized answer (1.7) holds.

Now we have to analyse the specific correlation contribution $R^{(c)}$ in (3.11). This is also a SL function whose PT expansion starts at the level of $(\alpha_{\rm s}\log)^2$.

2-Loop correlation. At two loops $R^{(c)}$ is given by the integral

$$R_A^{(c)} \;=\; \int d\omega_2(k_1,k_2)\left[U_A(k_1,k_2)-u_A(k_1)u_A(k_2)\right]\,, \qquad (3.20)$$

where $d\omega_2$ is the distribution for the correlated emission of two secondary partons off the primary $q\bar{q}$ system. Here $U(k)$ is an "inclusive" source function for the parent massive gluon. As was shown in [17], to properly reconstruct the two-loop Sudakov expression with the running physical coupling, this source has to be defined in terms of the single parton source function $u_A(k)=u_A(k_t,\eta)$ as

$$U_A(k_1,k_2) \;=\; u_A\left(\sqrt{k_t^2+m^2},\eta\right); \qquad k=k_1+k_2\,, \qquad m^2=k^2\,. \qquad (3.21)$$

Here η is the *rapidity* of the parent,

$$\sqrt{k_t^2+m^2}\,e^{\pm\eta} \;=\; k_{t1}e^{\pm\eta_1} \;+\; k_{t2}e^{\pm\eta_2}\,.$$

The distribution $d\omega_2$ is singular at $m^2=0$ when the two partons become collinear or one of them is much softer than the other. In particular, in the collinear limit the secondary partons obviously belong to the same angular region ($C_{\rm in}$ or $C_{\rm out}$) and then, as can be easily seen from (3.20), the combination of the sources in the square brackets vanishes thus regularizing the singularity. The same is true for the soft (energy ordered) two-gluon configuration, provided the gluons are in the same angular region. This results in a negligible subleading contribution $\mathcal{O}\left(\alpha_{\rm s}^2\log\right)$.

As we will see shortly, the only relevant logarithmically enhanced contribution emerges in the case when the two secondary partons belong to the complimentary angular regions, $C_{\rm in}$ and $C_{\rm out}$, so that the sources $u(k_1)$ and $u(k_2)$ become different and the cancellation gets broken. This is the configuration that gives rise to the non-global SL corrections, as was found by Dasgupta and Salam [6].

Given that there are no collinear logarithms, we can simplify (3.20) by taking strongly ordered parton energies,

$$k_{t2}\ll k_{t1}\ll Q\,. \qquad (3.22)$$

In this configuration the correlated 2-gluon distribution $d\omega_2$ in (3.20) reads [18]

$$d\omega_2(k_1,k_2) \;=\; 2C_F\,C_A\left(\frac{\alpha_{\rm s}}{\pi}\right)^2\prod_{i=1}^{2}\left(\frac{d\omega_i}{\omega_i}\frac{d\Omega_i}{4\pi}\right)\cdot C(\Omega_1,\Omega_2)\,. \qquad (3.23)$$

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The function C depends on parton angles and is given by the expression

$$\begin{aligned} C(n_1,n_2) &= \frac{(n\bar{n})}{(nn_1)(n_1n_2)(n_2\bar{n})} + \frac{(n\bar{n})}{(nn_2)(n_2n_1)(n_1\bar{n})} - \frac{(n\bar{n})}{(nn_1)(n_1\bar{n})}\frac{(n\bar{n})}{(nn_2)(n_2\bar{n})} \\ &= w_{n\bar{n}}(n_1)\,[\,w_{nn_1}(n_2) + w_{n_1\bar{n}}(n_2) - w_{n\bar{n}}(n_2)\,]\,, \end{aligned} \tag{3.24}$$

where

$$w_{ab}(c) \equiv \frac{(ab)}{(ac)(cb)}\,, \qquad (n_an_b) \equiv 1 - \cos\Theta_{ab}$$

and n, $\bar{n}$ stand for the direction 4-vectors of the primary partons and n_i, $i = 1, 2$ of the secondary gluons. The angular distribution (3.24) becomes singular when the gluon momenta $\vec{k}_1$ and $\vec{k}_2$ are parallel.

First we observe that in the soft limit (3.20) we can put $m^2 = 0$ in the source U to approximate

$$[\,U_A(k_1,k_2) - u_A(k_1)u_A(k_2)\,] \;\simeq\; u_A(k_1)\;[\,1 - u_A(k_2)\,]\,. \tag{3.25}$$

Let us first analyse the more complicated case of the associated distribution. Using the explicit expression for the source (3.6) we obtain two potential contributions:

$$\begin{aligned} &k_1 \in C_{\rm in},\; k_2 \in C_{\rm out}: \\ &\quad e^{-\nu v(k_1)}\left[1 - e^{-\nu v(k_2) - \nu_{\rm out}k_{2t}}\right] \Longrightarrow \min\{E_{\rm out}, VQ\} < k_{2t} < k_{t1} < VQ\,; \end{aligned} \tag{3.26a}$$

$$\begin{aligned} &k_1 \in C_{\rm out},\; k_2 \in C_{\rm in}: \\ &\quad e^{-\nu v(k_1) - \nu_{\rm out}k_{t1}}\left[1 - e^{-\nu v(k_2)}\right] \Longrightarrow VQ < k_{2t} < k_{t1} < \min\{E_{\rm out}, VQ\}\,. \end{aligned} \tag{3.26b}$$

We see that the only contributing region is that in (3.26a) where a harder gluon is close to the jet axis while a softer one contributes to the out-of-jet k_t-flow, provided $E_{\rm out} \ll VQ$. Therefore in (3.20) we can substitute

$$[U_{\rm 2ng}(k_1,k_2) - u_{\rm 2ng}(k_1)u_{\rm 2ng}(k_2)] \;\Rightarrow\; \theta_{\rm in}(k_1)\theta_{\rm out}(k_2)\,\vartheta(VQ - k_{1t})\,\vartheta(k_{2t} - E_{\rm out}) \tag{3.27}$$

and we get at two loops

$$R^{(c)}_{\rm 2ng} \;\Longrightarrow\; \frac{C_F}{C_A}\,\Delta^2(VQ, E_{\rm out}) \cdot F(\eta_0)\,. \tag{3.28}$$

Here $F(\eta_0)$ is given by the angular integral

$$F(\eta_0) = \int_{C_{\rm in}} \frac{d^2\Omega_1}{4\pi} \int_{C_{\rm out}} \frac{d^2\Omega_2}{4\pi}\, C(n_1,n_2) = \frac{\pi^2}{6} - {\rm Li}_2\left(e^{-4\eta_0}\right). \tag{3.29}$$

The same conclusion holds for the out-distribution:

$$[U_{\rm out}(k_1,k_2) - u_{\rm out}(k_1)u_{\rm out}(k_2)] \;\Rightarrow\; \theta_{\rm in}(k_1)\theta_{\rm out}(k_2)\,\vartheta(Q - k_{1t})\,\vartheta(k_{2t} - E_{\rm out})\,. \tag{3.30}$$

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Comparing this with (3.27) we see that this case reduces to setting $V = 1$ in (3.26), and we obtain

$$R_{\rm out}^{(c)} \implies \frac{C_F}{C_A}\,\Delta^2(Q,E_{\rm out})\cdot F(\eta_0)\,. \tag{3.31}$$

The result (3.28) is just the same expression that we derived for the away from jet distribution, except that what was Q there is now replaced by VQ.

This completes the proof of (1.7) with account of the 2-loop gluon correlations.

So we can write the integrated non-global distributions (2.3) with the SL accuracy as

$$\Sigma_{\rm out}(Q,E_{\rm out}) = e^{-\mathcal{R}_{\rm out}^{(1)}(Q,E_{\rm out})}\,e^{-\mathcal{R}_{\rm out}^{(c)}(Q,E_{\rm out})}\,, \tag{3.32a}$$

$$\Sigma_{\rm 2ng}(Q,V,E_{\rm out}) = e^{-\mathcal{R}_{\rm out}^{(1)}(VQ,E_{\rm out})}\,e^{-\mathcal{R}_{\rm 2ng}^{(c)}(Q,V,E_{\rm out})}\cdot\Sigma(Q,V)\,, \tag{3.32b}$$

where $\mathcal{R}_{\rm out}^{(1)}$ was defined in (3.15). We have shown in this section that with account of the 2-gluon correlation ($\mathcal{O}\left(\Delta^2\right)$)

$$\mathcal{R}_{\rm 2ng}^{(c)}(Q,V,E_{\rm out}) = \mathcal{R}_{\rm out}^{(c)}(VQ,E_{\rm out})\,. \tag{3.33}$$

In the following section we show that (3.33) hold at all orders in Δ.

4. Multi-gluon correlations in all orders

Since there is no collinear singularities in $R^{(c)}$, all SL contributions arise from ensembles of soft gluons with strongly ordered energies — the leading logarithmic soft approximation. As a result of the strong energy ordering, it is only the *hardest* of these soft gluons in the angular region $C_{\rm out}$ that contributes to $E_{\rm out}$ (since the contribution of *softer* gluons are negligible in the leading soft approximation).

Therefore, to collect these specific SL contributions to the non-global distributions (both the *out*- and *associated* distributions) it suffices to analyse multi-gluon systems with many gluons in $C_{\rm in}$ and a single one in $C_{\rm out}$. Let us denote the momentum of this gluon by q. Then, among the gluons $k_i \in C_{\rm in}$ it suffices to consider only those that are *harder* then q, since only such gluons may affect the radiation of q. Softer ones, $k_i \ll q$, don't contribute to the measurement due to real-virtual cancellations (except as power-suppressed corrections).

With account of multi-gluon effects the non-global correlation function $\mathcal{R}_A^{(c)}$ becomes much more involved. At the same time, the structure of its dependence on Q and V remains the same as in the 2-loop case. Indeed, the logarithmic integration in the energy of the hardest gluon k_1 in the $C_{\rm in}$ region is limited from above either by a pure phase space restriction $k_{10} < Q$ for the case of $\mathcal{R}_{\rm out}^{(c)}$ or, alternatively, by $k_{10} < VQ$ in the case of the associated distribution $\mathcal{R}_{\rm 2ng}^{(c)}$. Since this upper limit is the only place where the dependence on Q (QV) enters, we automatically derive the relation (3.33). Therefore it suffices to consider $\mathcal{R}_{\rm out}^{(c)}(Q,E_{\rm out})$. Hereafter we suppress the subscript and study

$$\mathcal{R}^{(c)}(Q,E_{\rm out}) \equiv \mathcal{R}_{\rm out}^{(c)}(Q,E_{\rm out}) = \mathcal{R}_{\rm 2ng}^{(c)}(Q,V=1,E_{\rm out})\,.$$

Generalisation. For the sake of generalisation the 2-loop result can be rephrased as follows. We take $k \in C_{\rm in}$ and $q \in C_{\rm out}$, $q \ll k$, and write ($\bar{\alpha}_s \equiv C_A \alpha_{\rm s}/\pi$)

$$\begin{aligned}\mathcal{R}^{(c)}(Q,E_{\rm out}) \;=\; & \int_{E_{\rm out}}^{Q} \frac{dk_0}{k_0} \int_{C_{\rm in}} \frac{d\Omega_k}{4\pi}\, \bar{\alpha}_s\, w_{n\bar{n}}(n_k) \times \\ & \times \left\{ r_{nn_k}(k_0,E_{\rm out}) + r_{n_k\bar{n}}(k_0,E_{\rm out}) - r_{n\bar{n}}(k_0,E_{\rm out}) \right\}, \qquad (4.1)\end{aligned}$$

where r_{ab} is the 1-loop radiator for the out-distribution generated by a colorless dipole ab with $a, b \in C_{\rm in}$:

$$r_{ab}(Q,E_{\rm out}) = \frac{2C_F}{C_A} \int_{E_{\rm out}}^{Q} \frac{dq_0}{q_0} \int_{C_{\rm out}} \frac{d\Omega_q}{4\pi}\, \bar{\alpha}_s\, w_{ab}(n_q)\,. \qquad (4.2)$$

In the large-N_c limit the structure of soft multi-gluon multiplication can be described in terms of the iterative procedure (see [7]):

$$W_{ab}(1,\dots,m) = w_{ab}(\ell)\, W_{a\ell}(1,\dots,\ell{-}1) W_{\ell b}(\ell{+}1,\dots,m)\,, \qquad W_{ab}(1) = w_{ab}(1), \quad (4.3)$$

where ℓ is one of m soft gluons emitted by the colour singlet dipole (ab). We choose ℓ to be the hardest of the secondary gluons. Following this path we arrive at the following generalisation:

$$\mathcal{R}^{(c)}_{ab}(Q,E_{\rm out}) = \int_{E_{\rm out}}^{Q} \frac{dk_0}{k_0} \int_{C_{\rm in}} \frac{d\Omega_k}{4\pi}\, \bar{\alpha}_s\, w_{ab}(n_k) \left\{ 1 - \frac{Z_{an_k}(k_0,E_{\rm out}) Z_{n_k b}(k_0,E_{\rm out})}{Z_{ab}(k_0,E_{\rm out})} \right\}, \qquad (4.4a)$$

where

$$-\ln Z_{ab}(E,E_{\rm out}) \;=\; r_{ab}(E,E_{\rm out}) + \mathcal{R}^{(c)}_{ab}(E,E_{\rm out})\,. \qquad (4.4b)$$

In (4.4) Z_{ab} is the generating function describing the gluon cascade which originates from the ab-dipole. We remark that the coupled equations (4.4) describing the large-angle (hedgehog) gluon configurations have a highly non-linear structure.

Once the equation is solved, we substitute the directions of the primary partons $n\bar{n}$ for ab to obtain

$$\mathcal{R}^{(c)}_{\rm out}(Q,E_{\rm out}) \;=\; \mathcal{R}^{(c)}_{n\bar{n}}(Q,E_{\rm out})\,. \qquad (4.5)$$

Recall that to obtain the all-order correlation function $\mathcal{R}^{(c)}_{\rm 2ng}(Q,V,E_{\rm out})$ for the associated distribution we simply have to substitute $Q \to VQ$ in (4.5).

The asymptotic regime. The behaviour of the answer in the asymptotic limit $\alpha_{\rm s} \ln(Q/E_{\rm out}) \gg 1$ ($\alpha_{\rm s} \ln(VQ/E_{\rm out}) \gg 1$) was analysed in [8]. For the partons a and b inside the same $C_{\rm in}$ cone and $\theta_{ab} \ll 1$

$$Z_{ab} \simeq h\left(\frac{\theta_{ab}^2}{2\theta_c^2}\right), \qquad \theta_c \simeq \lambda(\eta_0) e^{-\frac{c}{2}\Delta}\,, \qquad c \simeq 2.5 \qquad (4.6)$$

with c a universal number and λ a finite factor which depends on the opening angle. Here θ_c is a "critical angle" which depends on the SL parameter Δ defined in (3.16), and $h(x)$ is a universal function which is normalized, $h(0) = 1$, and decreases fast for large x as

$$h(x) \propto \exp\left(-\frac{c}{2}\ln^2 x\right).$$

When Δ is large, Z can be approximated by a theta-function

$$Z_{ab}(V, E_{\rm out}) \simeq \vartheta\left(\theta_c(\Delta) - \theta_{ab}\right).$$

The dependence on θ_0 becomes subleading in Δ, and we get an asymptotic estimate

$$\mathcal{R}^{(c)}(V, E_{\rm out}) \;=\; c \cdot \Delta^2 \,\left(1 + \mathcal{O}\left(\Delta^{-1}\right)\right). \qquad (4.7)$$

Let us remark that the $\mathcal{R}^{(1)}_{\rm out}$ (Sudakov) contribution to $\ln\Sigma$ is in this limit comparable with the subleading term in (4.7) which is geometry-dependent and could be computed by a numerical solution of the equation (4.4) or by the methods of [6, 12].

5. Conclusions

We considered the associated measurement of a global event shape variable V (e.g. thrust) and the away from jets transverse energy flow $E_{\rm out}$ (the "flow/shape" correlation [14]) in e^+e^- annihilation. We showed that the corresponding integrated distribution $\Sigma_{\rm 2ng}$ defined in (2.3b) factorizes into the product of the global distribution in V (2.1) and the (non-global) SL distribution in $E_{\rm out}$ (2.3a) taken at the rescaled energy VQ, as stated in (1.7). This result holds with DL+SL accuracy, including resummation of the SL terms in the ratio of the two characteristic scales, $(\alpha_{\rm s}\ln(VQ/E_{\rm out}))^n$.

This result hold also for other shape variables, in particular, for the broadening B, in which case the analysis of the global spectrum is more involved [5].

In the most interesting limit $VQ \gg E_{\rm out}$ the distribution Σ_{2ng} describes also the case when the V observable is measured only inside $C_{\rm in}$ (rather than in the whole phase space), as has been suggested in [14].

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